Opponents

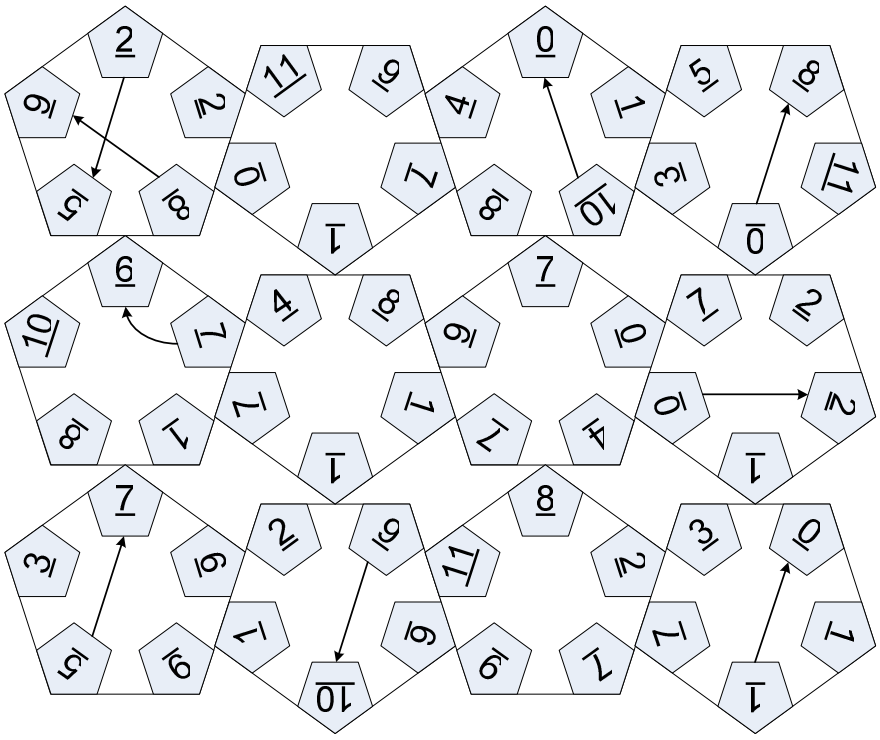
Everett Yip

Difficulty: Medium

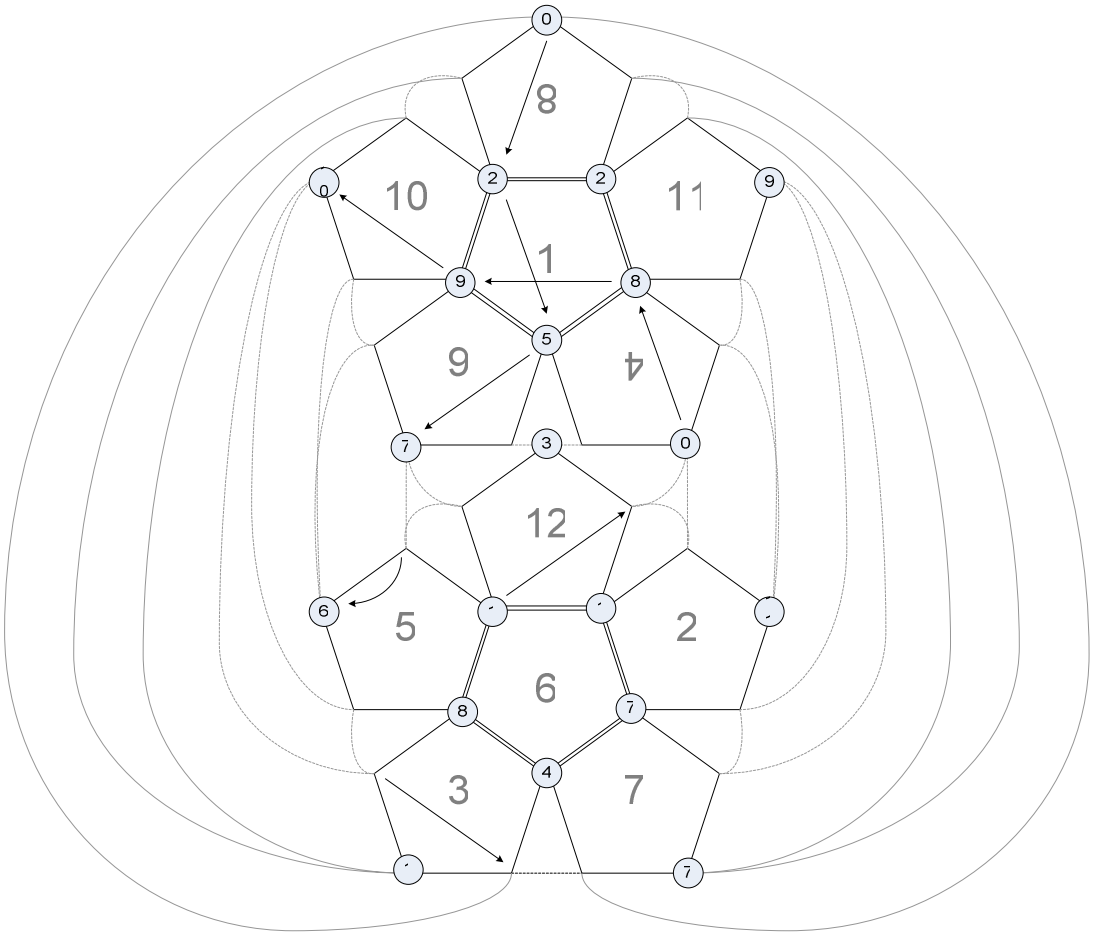
Dear students,

I'm sending you a model of something interesting I've found. You will have to assemble it yourself but I've numbered the vertices so it should be easy to match the pieces up. Notice what happens when you combine the poles. I'm going to investigate further.

- Prof. Stewart



You are presented a postcard with a short message from the professor indicating that he is sending you a model with numbered vertices which you need to assemble, followed by an array of 15 pentagons with outward pointing numbers at each vertex. The message is a pretty true description of the task required. The pentagons need to be arranged such that each edge attaches to the another side with the same values at their endpoints. The result is three-dimensional, with three pentagonal faces meeting at each vertex – i.e. a dodecahedron – as shown in the layout below.



In this diagram, each pentagonal face is numbered according to the appearance in the puzzle. Unique vertices are marked with circled numbers containing their label value. Gray lines connect vertices that, when assembled in three dimensions, join into the same vertex.

The arrows that are visible on the individual faces connect to form a single chain when the dodecahedron is constructed. There is a starting vertex – with only an arrow leaving it – a terminating vertex – with only an arrow entering it – and 8 intermediate vertices – with one entering and one leaving arrow. This path contains 10 unique nodes – i.e. none used twice – with the following values:
$1 \rightarrow 0 \rightarrow 8 \rightarrow 9 \rightarrow 10 \rightarrow 0 \rightarrow 2 \rightarrow 5 \rightarrow 7 \rightarrow 6$
This leaves 10 extra nodes unconnected.

Notably, every vertex in the path has, as a point diametrically opposite it on the dodecahedron, an unconnected vertex. This interesting arrangement clues the final step required to extract the message in addition to the message which suggests to "Notice what happens when you combine the poles". By adding the labeled value from the opposite vertex to each one in path, series of numbers is arrived at.

For example, the first vertex in the path has value 1. The node diametrically opposed has the value 2. So the new value of the first vertex in the path is 3. Repeating for the entire sequence produces the following result:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Position Along Path**
1	0	8	9	10	0	2	5	7	6	Vertex Value
2	1	8	7	11	3	1	4	7	9	Opposite Vertex Value
3	1	16	16	21	3	3	9	14	15	New Value (sum)

The new values can then be converted to letters using the standard encoding of 1=A, 2=B, etc.

This produces the letters C A P P U C C I N O

The answer is Cappuccino.

Hints:

- each vertex must be consistent
- follow the path
- there is also an opposite path
- nothing is unused
- bring the ends together